CLASS OF SOLUTIONS FOR FLOW NEAR A STAGNATION POINT

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This paper considers a class of solutions for flow of a perfect gas near the stagnation point on a two-dimensional obstacle, where the flow is rotational far upstream from the obstacle. It is shown that the potential flow near the stagnation point is a special case of this class of solutions. Solutions accounting for the rotationality of the outer flow are obtained for flow in the mixing layer with an obstacle, and these solutions differ appreciably from the analogous Jimenez solution for potential flow near the stagnation point on a two-dimensional obstacle.

We consider two-dimensional flow of an incompressible fluid flowing normal to a two-dimensional obstacle of infinite length, in a rectangular coordinate system xOy, where x is directed along the obstacle, and y is normal to it. The flow region is bounded by the surface $y = 0$ at the obstacle, with stagnation point $x = y = 0$, and the section y_∞, where the effect of the obstacle is negligible, or where the well-known law is assumed for deformation of the velocity profile of the exterior flow by the obstacle. The Jimenez solution is well known, describing viscous flow near the stagnation point for external potential flow [1]. We show that this problem can also be solved for more complex flows, when the outer flow of a perfect fluid is rotational. The presence of vorticity at the outer boundary of the viscous mixing layer at the obstacle leads in this case to certain differences from the Jimenez solution.

1. Let the stream of perfect fluid incident on the obstacle be uniform, with speed V_∞ at $y = y_\infty$. Putting $v/V_\infty = -F(y/y_\infty)$, we obtain from the continuity and vorticity-transfer equations

$$u/V_\infty = F'x/y_\infty;\quad FF''' - F'F'' = 0, \tag{1.1}$$

where v and u are the velocity components normal to and tangential to the obstacle, respectively.

The solution of Eq. (1.1) with boundary conditions $y/y_\infty = 0$, $F = 0$; $y/y_\infty = 1$, $F = 1$, $F' = 0$ has the form

$$F = \sin\frac{\pi}{2}\frac{y}{y_\infty}. \tag{1.2}$$

It follows from Eq. (1.2) that

$$v = -V_\infty \sin\frac{\pi}{2}\frac{y}{y_\infty};\quad u = \frac{\pi}{2}\frac{V_\infty}{y_\infty}x\cos\frac{\pi}{2}\frac{y}{y_\infty}. \tag{1.3}$$

It can be shown that this solution corresponds to flow far from the obstacle when the vorticity depends linearly on the coordinate x:

$$\Omega(1) = \frac{\pi^2}{4}\frac{V_\infty}{y_\infty^2}x.$$

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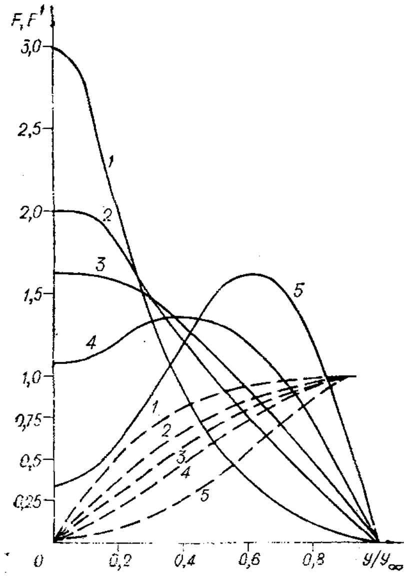

Fig. 1

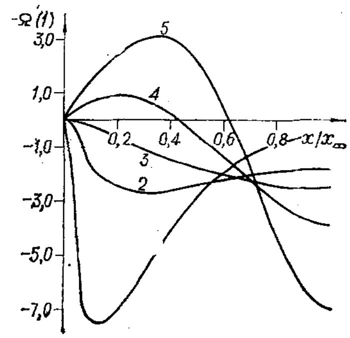

Fig. 2

2. There are cases where it is of interest to analyze flow near the stagnation point, when the outer flow incident on the obstacle is not uniform, but varies with x according to some law different from linear. We shall show that there exists a solution of type (1.3) for a specific class of functions describing the variation of the outer flow velocity with x. We seek a solution for the normal velocity component in the form

$$v/V_0 = -F(y/y_\infty)\,\Phi'(x/y_\infty), \tag{2.1}$$

where V_0 is the characteristic velocity at the section $y = y_\infty$. Then for the velocity component tangential to the obstacle we have

$$u/V_0 = F'(y/y_\infty)\Phi(x/y_\infty). \tag{2.2}$$

From the vorticity-transport equation, and taking account of Eqs. (2.1) and (2.2), we have

$$\frac{\Phi\Phi''' - \Phi'\Phi''}{\Phi\Phi'} = \frac{FF''' - F'F''}{FF'} = \text{const.} \tag{2.3}$$

Taking the constant in Eq. (2.3) equal to zero, we obtain two equations of type (1.1) in Φ and F. The solution of these for boundary conditions,

$$\begin{aligned} &y/y_\infty = 0,\ F = 0;\ y/y_\infty = 1,\ F = 1,\ F' = 0;\\ &x/y_\infty = 0,\ \Phi = 0;\ x/y_\infty = x_\infty/y_\infty,\ \Phi = 1.\\ &\Phi' = 0 \end{aligned} \tag{2.4}$$

has the form

$$F = \sin\frac{\pi}{2}\frac{y}{y_\infty};\quad \Phi = \sin\frac{\pi}{2}\frac{x}{x_\infty}. \tag{2.5}$$

It follows from Eq. (2.5) that

$$\begin{aligned} v &= -\frac{\pi}{2}\frac{y_\infty}{x_\infty}V_0 \sin\frac{\pi}{2}\frac{y}{y_\infty}\cos\frac{\pi}{2}\frac{x}{x_\infty};\\ u &= \frac{\pi}{2}V_0\cos\frac{\pi}{2}\frac{y}{y_\infty}\sin\frac{\pi}{2}\frac{x}{x_\infty}. \end{aligned} \tag{2.6}$$

If we assume that the velocity V_∞ has the same value on the axis of symmetry at $y = y_\infty$ as in the previous case, it follows from Eq. (2.6) that $V_0 = V_\infty/\Phi'(0)$. The solution obtained represents flow with vorticity equal to

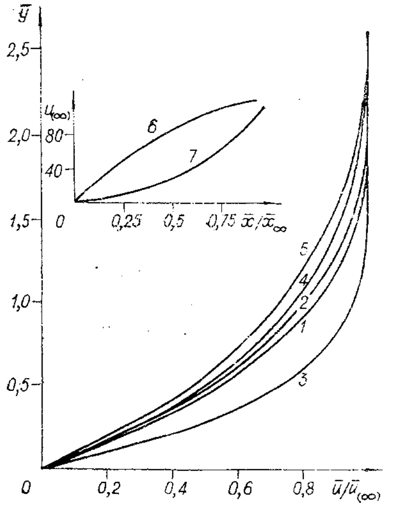

Fig. 3

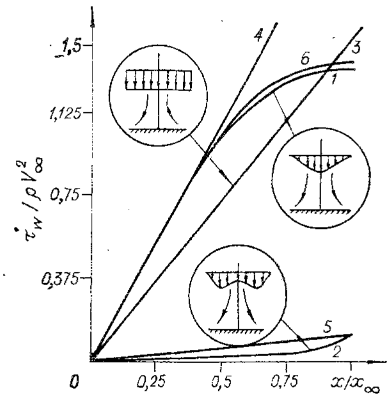

Fig. 4

$$\Omega(1)=\left[1+\left(\frac{x_\infty}{y_\infty}\right)^2\right]\frac{y_\infty}{x_\infty}\frac{\pi}{2}\frac{V_\infty}{y_\infty}\sin\frac{\pi}{2}\frac{x}{y_\infty},\quad 0\leqslant x\leqslant x_\infty.$$

3. Taking a nonzero value for the constant in Eq. (2.3), we obtain the following two equations determining ideal flow near a stagnation point:

$$FF'''-F'F''-AFF'=0;\ \Phi\Phi'''-\Phi'\Phi''-A\Phi\Phi'=0. \tag{3.1}$$

The solution of these for boundary conditions (2.4) was obtained numerically on the BÉSM-4 computer by a Runge–Kutta method, to an accuracy of 10^{-5}, for a number of values of A. The search for an additional boundary condition was carried out using Newton's method, i.e., $f[F''(1)] = |F(0)| < \varepsilon$, where $\varepsilon = 10^{-4}$ is the allowable error in applying the boundary condition. The results of the solution are presented for the functions F and F' (broken lines) in Fig. 1 (curves 1-5 are for A = 20, 5, 0.1, −5 and −14, respectively). The functions Φ, Φ' have a similar form.

We now find the range of variation of parameter A for which Eqs. (3.1) have a solution. We consider an equation for $F(y/y_\infty)$. Using the condition $F'(1) = 0$, this can be written in the form

$$F'^2=\left(C-\frac{A}{2}\right)(F^2-1)+AF^2\ln F. \tag{3.2}$$

It can be seen from the last equation that a necessary condition for obtaining real roots of the function F' is the condition $(C - A/2) \le 0$ for negative A, i.e., $C \le A/2$, $A \le 0$. Since with $A = 0$ $C = -\pi^2/4$, Eq. (3.2) has a solution for all negative A. For $A > 0$ the region of positive roots of F' is given by the condition

$$|AF_m^2\ln F_m|\leqslant|(C-A/2)(F_m^2-1)|, \tag{3.3}$$

where F_m corresponds to F with $\max(F^2\ln F)$. Taking into account Eq. (3.3), we obtain $-\infty < C < |A|/5$.

We now find the solution of Eq. (3.2) with $C = A/2$, $A < 0$. In this case

$$F=\exp\left[\frac{A}{4}(1-y/y_\infty)^2\right],$$

i.e., the solution for boundary conditions (2.4) is valid only for $A \to -\infty$, and therefore the condition $C = A/2$ holds approximately only when $A \ll 0$.

The solution of Eqs. (3.1) corresponds to outer flow vorticity Ω (1) (Fig. 2, curves 1-5 with A = 20, 5, 0.1, −5, and −14, respectively):

$$\Omega(1) = -\frac{V_0}{y_\infty} F''(1)\left[\Phi\left(\frac{x}{y_\infty}\right) + \frac{\Phi''(x/y_\infty)}{F''(1)}\right], \quad -\infty < F''(1) < |A|/5,$$

where $x_\infty = y_\infty$; and V_0 is the characteristic velocity at the outer boundary $y = y_\infty$. Denoting V_∞ as the maximum velocity at $y = y_\infty$, we obtain that $V_0 = V_\infty/\max[\Phi'(x/y_\infty)]$.

For $A \ll 0$ an expression for the vorticity in the outer flow (at $x_\infty = y_\infty$) can be written approximately in the form

$$\Omega(1) \simeq \frac{V_\infty}{y_\infty}\sqrt{2e|A|}\left[1 + \frac{A}{4}\left(1 - \frac{x}{y_\infty}\right)^2\right]\exp\left[\frac{A}{4}\left(1 - \frac{x}{y_\infty}\right)^2\right].$$

To classify the solutions obtained we note that the parameter A is a shape factor for the velocity profile of the flow incident on the obstacle. For $A \geq 0$ the maximum velocity in the incident flow coincides with the axis of symmetry, and for $A < 0$ the maximum velocity is displaced away from the symmetry axis.

4. We now investigate the effect of a transfer velocity component U_∞ at the outer boundary $y = y_\infty$ of the region of interaction between a perfect fluid and an obstacle. Let $A = 0$ and $F(1) = 1$; $F'(1) = B$, where $U_\infty = BV_\infty\Phi(x)$. Then for $F(0) = 0$ we obtain the following equation for $F(y/y_\infty)$:

$$F' = \pm\sqrt{B^2 - C}\sqrt{1 - \frac{C}{C - B^2}F^2}, \quad C = F''(1). \tag{4.1}$$

If $C/(C - B^2) > 0$, then for $C \leq 0$ F' has real roots for any B. Since $\sqrt{-C} = \pi/2$ at $B = 0$, in the intervai $[-\pi/2, 0]$ the solution of Eq. (4.1) has the form

$$F\sin\sqrt{-C} = \sin\sqrt{-C}y/y_\infty,$$

and, as follows from the condition $F(1) = 1$, the quantity $\sqrt{-C}$ depends on B as follows:

$$B = \pm\sqrt{-C}\operatorname{ctg}\sqrt{-C}. \tag{4.2}$$

It can be seen from Eq. (4.2) that $\sqrt{-C} \to 0$ as $B \to \pm 1$. The last value at $B = 1$ corresponds to flow of a potential fluid near the stagnation point on the obstacle: $F = y/y_\infty$; $v = -V_\infty y/y_\infty$; $u = V_\infty x/y_\infty$, and for $B = -1$ it corresponds to potential flow away from an obstacle.

Assuming $C/(C - B^2) < 0$, we obtain for $C > 0$, $B^2 \geq C$:

$$F\operatorname{sh}\sqrt{C} = \operatorname{sh}\sqrt{C}y/y_\infty,$$

where $\sqrt{C}$ is connected with B by the relationship

$$B = \pm\sqrt{C}\operatorname{cth}\sqrt{C}.$$

The solutions obtained for $A = 0$ correspond to flow at the outer boundary $y = y_\infty$ with vorticity equal to

$$\Omega(1) = -V_\infty Cx/y_\infty^2; \quad v(y = y_\infty) = -V_\infty = \text{const};$$
$$\Omega(1) = \left[1 - C\left(\frac{2}{\pi}\frac{x_\infty}{y_\infty}\right)^2\right]\frac{y_\infty}{x_\infty}\frac{\pi}{2}\frac{V_\infty}{y_\infty}\sin\frac{\pi}{2}\frac{x}{x_\infty};$$
$$v(y = y_\infty) = -V_\infty\cos\frac{\pi}{2}\frac{x}{x_\infty},$$

where $C = F''(1)$, and $C < 0$ $\Omega > 0$, and $C > 0$ $\Omega < 0$.

Similar relationships can be obtained also for flow with a shape factor $A \neq 0$.

5. The cases considered with flow of a perfect fluid near the stagnation point require specific conditions as regards the flow vorticity at a given distance from the obstacle in order to be realized in practice. It is clear that flow vorticity due to the action of viscous forces can be obtained by choosing the shape of the channel in which the flow is formed, and also by planned mixing of the flow in the channel or in the surrounding medium. Examples of this kind of flow are flow in converging and diverging channels, secondary flows, including secondary flow behind a body washed in a longitudinal direction [1, 2], and flows

in jets with uniform and nonuniform velocity profiles at the nozzle exit [3]. In the last case one can identify a number of physical analogs of the solutions obtained, well known from practical use of the effects of interaction of a jet with obstacles. For example, the solution when the profile shape factor of the flow incident on the obstacle $A > 0$ corresponds to the case of interaction of a subsonic jet with an obstacle within the main section of the jet, while the case $A < 0$ corresponds to interaction in the initial section, where the obstacle deforms the velocity profiles of the jet at the nozzle exit, i.e., for small distances between the obstacle and the nozzle exit [4, 5].

We now examine the effect of profile shape of the flow incident on the obstacle on the character of the flow near the obstacle. To do this we consider viscous flow in a layer near an obstacle, when the outer flow is described by velocity-profile shape factors which differ in magnitude and sign. We take the Navier–Stokes equations as the initial system of equations determining the viscous flow. We formulate the boundary conditions. It follows from the solution for a perfect fluid that the velocity gradient at the stagnation point at $x_\infty = y_\infty$ is

$$\left(\frac{\partial v}{\partial y}\right)_{y=0} = -\frac{V_\infty}{y_\infty} F'(0) \frac{\Phi'(x/y_\infty)}{\max\left[\Phi'(x/y_\infty)\right]},$$

where V_∞ is the maximum velocity at the section $y = y_\infty$ (for $A \geq 0$ the maximum velocity is on the axis of symmetry $x = 0$ and for $A < 0$ it is displaced outwards from the symmetry axis).

Using the notation $\beta = V_\infty F'(0)/y_\infty$, and taking into account that the variation of the velocity normal to the obstacle is nearly linear at small distances from the obstacles ($y \to 0$), we can adopt the relationship

$$\bar{v}(\infty) = -\Phi'(\bar{x}/\bar{y}_\infty)\,\bar{y}/\Phi'_m; \quad \bar{u}(\infty) = \Phi(\bar{x}/\bar{y}_\infty)\,\bar{y}_\infty/\Phi'_m,$$

as an outer condition for viscous flow near the obstacle, where $\bar{v}, \bar{u} = v, u/\sqrt{\beta\nu}$; $\bar{x}, \bar{y} = (x, y)\sqrt{\beta\nu}$ (ν is the coefficient of kinematic viscosity);

$$\Phi'_m = \max\left[\Phi'(\bar{x}/\bar{y}_\infty)\right].$$

As the boundary condition at the wall we use the condition for nonslip of the fluid at the wall: $\bar{u}(0) = \bar{v}(0) = 0$.

We represent the function $\Phi'(\bar{x}/\bar{y}_\infty)/\Phi'_m$ in the form of a power series containing only even powers of x (the Blasius series [1]):

$$\frac{\Phi'(\bar{x}/\bar{y}_\infty)}{\Phi'_m} = \sum_{n=0}^{\infty} (-1)^n a_{2n}\bar{x}^{2n} = \sum_{n=0}^{\infty} (-1)^n a_{2n}\left(\frac{\beta}{\nu} y_\infty^2\right)^n \left(\frac{x}{y_\infty}\right)^{2n}$$

and look for a solution for $\bar{v}, \bar{u}$ in the viscosity mixing layer near the obstacle ($0 \leq \bar{y} \leq \infty$) in the form of the appropriate series:

$$\bar{v} = -\sum_{n=0}^{\infty} (-1)^n a_{2n}\bar{x}^{2n} f_{2n}(\bar{y});$$

$$\bar{u} = \sum_{n=0}^{\infty} (-1)^n \frac{a_{2n}\bar{x}^{2n+1}}{(2n+1)} f'_{2n}(\bar{y}),$$

where a_{2n} are known coefficients for series expansion of $\Phi'(\bar{x}/\bar{y}_\infty)/\Phi'_m$: and $f_{2n}(\bar{y})$ are functions of $\bar{y}$ subject to definition.

The Navier–Stokes equations in the form of the vorticity-transport equation give the following equation for $f_{2n}(\bar{y})$:

$$\sum_{n=0}^{\infty}{}'(-1)^n a_{2n}\bar{x}^{2n-3}\left[\frac{\bar{x}^4}{2n+1} f_{2n}^{\mathrm{IV}} + 4n\bar{x}^2 f''_{2n} + 4n(2n-1) \times \right.$$
$$\left. \times (n-1) f_{2n}\right] + \sum_{n,k=0}^{\infty} (-1)^{n+k} a_{2n}a_{2k}\bar{x}^{2(n+k)-1}\left\{\bar{x}^2\left[\frac{f_{2n}f'''_{2k}}{2k+1} - \right.\right. \tag{5.1}$$
$$\left.\left. -\frac{f'_{2n}f''_{2k}}{(2n+1)k}\right] + 2k\left(f_{2n}f'_{2k} - \frac{2k-1}{2n+1} f'_{2n}f_{2k}\right)\right\} = 0.$$

Equating coefficients of identical powers of $\bar{x}$, we obtain a system of ordinary differential equations of fourth order to determine the functions f_0, f_2, f_4,.... The boundary conditions for the functions f_{2n} are obtained from the condition $\bar{y} = 0$ and $\bar{y} \to \infty$:

$$\bar{y} = 0,\ f_{2n} = f'_{2n} = 0; \quad \bar{y} \to \infty,\ f'_{2n} = 1,\ f''_{2n} = 0. \tag{5.2}$$

The system of equations (5.1) with boundary conditions (5.2) was solved numerically by a method of successive approximations, using a Runge–Kutta method to an accuracy of 10^{-5} on a BÉSM-4 computer. Since each of the n equations contains n unknown functions and their derivatives, we assume in the first approximation that all the functions and their derivatives, except f_0, are zero. Thus, for n = 0, we obtain the Jimenez equation, whose solution is known [1].* In solving the second equation (n = 1) we use the results for the function f_0 and its derivatives at each step of integration, assuming all the f_{2n} (n = 2, 3, ...) and their derivatives to be zero. The assignment of f_0, f_0', f_0'', etc., in the form of stepwise functions at each integration step does not introduce appreciable error and does not affect the convergence process to the conditions with $\bar{y} \to \infty$, and this was checked by varying the integration step. The optimum step, $\Delta\bar{y} = 0.05$, was chosen from the condition $|f_{2n}(\infty, \Delta\bar{y}) - f_{2n}(\infty, 2\Delta\bar{y})| < 10^{-5}$. The solution for the subsequent equations with n = 2,3... was carried out analogously. Using the results of the first approximation, we proceeded to the next approximation, using the above-mentioned scheme. The number of approximations was determined from the condition $|f_{2n}(\infty, t+1) - f_{2n}(\infty, t)| < 10^{-4}$, where t, t + 1 are the numbers of the approximations. The search for additional boundary conditions necessary to begin the computation was carried out, as in the case when solving the ideal problem, by Newton's method, in such a way that $F_1[f'_{2n}(\infty)] = |f'_{2n}(\infty) - 1| < \varepsilon$; $F_2[f''_{2n}(\infty)] = |f''_{2n}(\infty)| < \varepsilon$, where $\varepsilon = 10^{-4}$ is the allowable error in applying the boundary conditions.

The initial data for the computation were as follows: $V_\infty = 8$ m/sec; $x_\infty = y_\infty = 0.05$ m; $\nu = 1.5 \cdot 10^{-5}$ m^2/sec. In practice, the convergence of the series in Eq. (5.1), as later calculations showed, was quite good, and therefore only a finite number of terms (n, k $\leq$ 4) of the above series was required for approximate solution of the problem of viscous flow near a stagnation point. Figure 3 shows the profiles of the velocity component $\bar{u}$ in the boundary layer over an obstacle with A = 5.0, with $\bar{x}/\bar{x}_\infty = 0.125$ (curve 1); A = 5.0 with $\bar{x}/\bar{x}_\infty = 0.75$ (curve 2); and A = −14 with $\bar{x}/\bar{x}_\infty = 0.75$ (curve 3). The figure also shows results corresponding to the Jimenez solution [1] for flow near a stagnation point on a two-dimensional flat plate and uniform outer incident flow (curve 4). Curve 4 corresponds also to the Jimenez solution for a cylinder of radius $R = 2y_\infty/F'(0)$, $F'(0) = 2$ and $R - 2y_\infty/(F'(0)a_0)$, $F'(0) = 0.35$, $a_0 = 0.218$ for $\bar{x}/\bar{x}_\infty = 0.125$. Curve 5 corresponds to the Jimenez solution for a cylinder of radius $R = 2y_\infty/F'(0)$, $F'(0) = 2$ for $\bar{x}/\bar{x}_\infty = 0.75$. The difference in the velocity profiles for the types of flow considered near the stagnation point of an obstacle and a cylinder with uniform outer flow results from the different nature of the velocity distribution over the obstacle in the outer flow (curves 6 and 7 are for outer-flow velocity-profile shape factors A = 5.0 and −14.0, respectively). Near the stagnation points ($\bar{x}/\bar{x}_\infty = 0.125$) the velocity profiles in both cases considered practically coincide with the Jimenez profile (the part of the outer flow with constant acceleration). Further from the stagnation point there is an increase in the gradient of velocity $\bar{u}$ in the direction normal to the obstacle for outer flow with a peripheral maximum velocity A < 0 (the accelerated section of the outer flow is along the obstacle) and a reduced velocity gradient $\bar{u}$ in the direction normal to the wall for flow with a central maximum velocity A $\geq$ 0 (the deceleration section for outer flow over an obstacle). Figure 4 shows the distribution of friction on the obstacle wall for incident flow with a central maximum velocity (curve 1) and a peripheral maximum velocity (curve 2):

$$\frac{\tau_w}{\rho V_\infty^2} = a_0 F'(0) \frac{1}{\mathrm{Re}} \frac{\partial u}{\partial y}\bigg|_{\bar{y}=0}; \quad \mathrm{Re} = \frac{V_\infty y_\infty}{\nu}.$$

Curves 3–5 correspond to the Jimenez solution for uniform outer flow incident on a two-dimensional obstacle:

$$\frac{\tau_w}{\rho V_\infty^2} = 1.23259 \frac{1}{\mathrm{Re}} y_\infty^2 \frac{\beta}{V_\infty} \left(\frac{\beta}{\nu}\right)^{1/2} \bar{x}/\bar{x}_\infty.$$

The value $\beta = (\pi/2)V_\infty/y_\infty$ (curve 3) corresponds to a uniform stream located above the obstacle at the given distance y_∞. The value $\beta = V_\infty F'(0)/y_\infty = \beta_1$ (curve 4) corresponds to flow over the obstacle of a uniform stream with a velocity gradient at the stagnation point equal to the velocity gradient at the stagnation point in flow of an outer stream with a central maximum over a two-dimensional obstacle. The value $\beta = V_\infty a_0 F'(0)/y_\infty = \beta_{2*}$ (curve 5) corresponds to flow over the obstacle of a uniform stream with a velocity

* For $a_0 \neq 1$ a correction is applied to the value of β: $\beta\,(a_0 \neq 1) = V_\infty F'(0) a_0/y_\infty$.

gradient at the stagnation point equal to that at the stagnation point in flow over the obstacle of an outer stream with a peripheral velocity maximum. In the last two cases, as is shown by the expressions derived for β, the obstacle must be brought close to the stream, to a distance $0.785\ y_\infty$ (with the condition that the flow at this distance remains undisturbed), and, conversely, must be withdrawn from the stream to a distance $20.6\ y_\infty$, where y_∞ corresponds to the distance from the obstacle at which the assumed nonuniformity of the external stream is maintained. Finally, curve 6 corresponds to the Jimenez solution for a cylinder of radius $R = 2y_\infty F'(0)$ ($A = 5.0$). In the case $A = -14.0$ the Jimenez solution for a cylinder of radius $R = 2y_\infty/(F'(0)\alpha_0)$ practically coincides with the Jimenez solution for a two-dimensional obstacle (curve 5).

The results shown in Fig. 4 allow several special features to be identified in the nature of the friction distribution near the stagnation point on a two-dimensional obstacle, located normal to an external nonuniform flow:

1. There is a strong dependence of friction on velocity gradient at the stagnation point for all types of outer flow examined. The velocity gradient at the stagnation point is determined by the conditions of flow stagnation near the obstacle, and, as is shown in the present analysis, by the conditions for formation of the flow at a given distance from the obstacle.

2. For flow with a central maximum velocity ($A \geq 0$) the value of the velocity gradient at the stagnation point, $\beta = V_\infty F'(0)/y_\infty$, gives the friction at the wall near the stagnation point, equal to the friction calculated for a uniform flow incident on a two-dimensional obstacle and a circular cylinder of radius $R = 2y_\infty/F'(0)$. In the latter case the friction on a two-dimensional obstacle, washed by a nonuniform flow, and on the surface of a cylinder, washed by a uniform flow, practically coincide for the whole range of x variation considered. This follows from similarity in the law for velocity distribution in the outer flow over the surface of the embedded bodies. The analogy between flow over a cylinder and flow normal to an obstacle, based on similarity of velocity profiles of the outer flow over the surface of embedded bodies and on the equality of corresponding velocity gradients at the stagnation point, suggests the need, as has already been mentioned, to reduce the initial distance y_∞ (for $A = 5.0$) between the obstacle and the undisturbed outer flow to the value of $(\pi/4)y_\infty$. If we require that the value of y_∞ remains unchanged, then, as follows from comparison of the expression for the velocity gradient at the stagnation point on the obstacle due to uniform and nonuniform external flows, we find that the given condition is satisfied only for one value $F'(0) = \pi/2$, i.e., for a sinusoidal variation of velocity of the outer flow over the obstacle, when the shape factor of the outer-flow velocity profile $A = 0$. For nonzero values of A, with the condition that the distance y_∞ remains constant, we can use the analogy between flow of a nonuniform stream over a two-dimensional obstacle and over a body whose shape differs from a circular cylinder. In fact, as follows from Fig. 1, an increase in the value of A leads to an increase in velocity gradient at the stagnation point and to a more rapid "filling out" of the velocity profile of the outer flow over the obstacle. The same effect is observed for flow of a uniform stream over an elliptic cylinder in the longitudinal direction (parallel to the major axis).

For flow with a peripheral velocity maximum ($A < 0$) we observe a sharp reduction in the friction at the wall (by an order of magnitude) in comparison with the stream having a central velocity maximum. The friction distribution in this case, as in the case of outer flow with a central velocity maximum, follows the velocity distribution law in the outer flow over the obstacle, and it is therefore natural to find a large discrepancy between the results obtained and the Jimenez data for a uniform flow washing a circular cylinder or an obstacle located normal to the flow.

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